ST HILDA'S
ANGLICAN SCHOOL FOR GIRLS INC.

Time:	16 minutes
Marks:	16
Total:	42

Mathematics Methods 3&4

Response Test 3 – Calculator Free

(Thursday August 19th)

ClassPad calculators are NOT permitted.

Formulae Sheet is permitted.

Name: _____

1. [1 & 2 = 3 marks]

(a) Use base 10 logarithms to solve the equation $2^{3x} = 5$ exactly.

(b) Solve the equation $5\log_2(3x-1) = 15$ giving your answer in simplest form.

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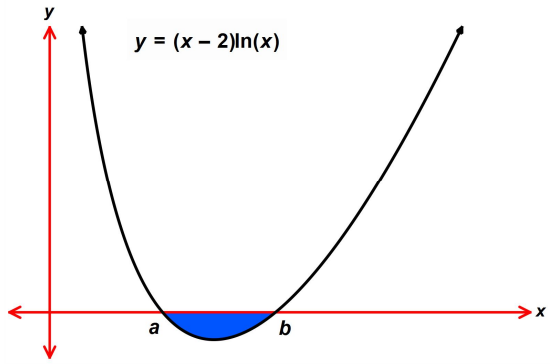
2. [2 & 2 = 4 marks]

(a) Find $\frac{dy}{dx}$ in simplest form if $y = \ln(4\sin(3x))$.

(b) Find the exact value of k if $\int_1^7 \frac{2}{4x-3}dx = \ln(k)$

3. [2, 4 & 3 = 9 marks]

The curve with equation $y = (x-2)\ln(x),\ x > 0$ is shown on the axes to the right.



(a) The graph has x-intercepts at $x = a$ and $x = b$. Determine the value of a and b.

(b) Find the equation of the tangent to the curve at the point where $x = b$.

(c) The area of the shaded region between the curve and the x-axis is given by the definite integral $\int_c^d (x-2)\ln(x)\,dx$ which has the positive value of $\ln\left(4e^{\frac{5}{4}}\right)$.

(i) State the value of c.

(ii) The area of the shaded region $\ln\left(4e^{\frac{5}{4}}\right)$ can be expressed in the form $p\ln(q)+r$. Find the exact value of the rational constants p, q and r.

End of Resource Free

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ANGLICAN SCHOOL FOR GIRLS INC.

Time: 28 minutes

Marks: _____ / 26 marks

Mathematics Methods 3&4

Response Test 3 – Calculator Assumed

(Thursday August 19th)

Half an A4 page of notes and ClassPad calculators are permitted.

Formulae Sheet is permitted.

Name: _____

4. [1, 2, 2 & 1 = 6 marks]

The number n of patients with a disease t weeks after commencing a course of treatment is modelled by $n(t) = 50 + 50\ln(e - t)$, $0 \leq t \leq b$.

(a) How many patients have the disease initially?

(b) To the nearest day, how many days after commencing treatment are there 20 patients with the disease?

(c) Correct to the nearest whole number, what is the rate of change of n when t = 1·5

(d) The model ceases to be valid when all patients are cured. Determine the exact value of b.

5. [1, 2 & 5 = 8 marks]

A company has ten telephone lines. At any instant, the probability that any particular line is engaged (in use) is $\frac{1}{5}$. Let X = the number of the ten telephone lines that are free.

(a) State the type of probability distribution that X follows including the values of relevant parameters.

(b) **(i)** State the expected number of free (not in use) telephone lines.

(ii) Find the variance of the number of free telephone lines.

(c) Calculate, correct to 3 decimal places, the probability that

(i) 4 of the lines are engaged

(ii) at least 4 lines are free

(iii) at least 6 lines are free if at least 4 lines are free

6. [3, 1 & 2 = 6 marks]

The temperature, X degrees Celsius inside a refrigerator has been found to have a probability density function $f(x) = \begin{cases} \frac{x}{k\pi}\sin\left(\frac{x}{4}\right) & , \ 0 \le x \le 4\pi \\ 0 & , \ \text{elsewhere} \end{cases}$ where k is a constant.

(a) Find

(i) the value of k

(ii) the probability that the refrigerator's temperature is between $5^{\circ}C$ and $12^{\circ}C$

(b) Calculate the exact mean temperature inside this refrigerator.

(c) Calculate the standard deviation of the temperature inside this refrigerator correct to three decimal places.

7. [3, 1 & 2 = 6 marks]

The intelligence quotient or IQ, as measured by IQ tests, is a normally distributed random variable with mean of 100 and standard deviation of 15.

There are currently 10000 members of the West Coast Eagles.

(a) How many of the 10000 members of the West Coast Eagles would be expected to have an IQ that is

(i) between 90 and 120?

(ii) over 130?

(b) Find the 0·6 quantile of IQ's of the members of the West Coast Eagles.

(c) If four of the 10000 members of the West Coast Eagles are randomly selected, what is the probability that exactly one of the four has an IQ over 130?

End of Resource Rich